Sensitivity Analysis of Physiographic Parameters By Kinematic Wave Equation

Mr. Md. Abu Salek*
Muhammad Abul Kalam Azad**
Jannatun Nayeem***

Abstract

Sensitivity analysis of physiographic parameters describes the system behavior in terms of mathematical equations representing the relationships between system state, input and output. The channel flow routing and overland flow routing are depending on different kind of flow parameters. In this paper, the channel flow routing and overland flow routing for different kind of channel and overland flow parameters are discussed. The computer program for flow routing is developed in Lahey ED Developer. The results showed that the sensitivity of physiographic parameters through lumped kinematics wave models and found that the overland roughness and overland slope are more sensitive than other physiographic parameters of overland and channel flows.

Key words: Channel and overland flow routing, physiographic parameters, Kinematics wave Model, Sensitivity test.

1. Introduction

Rainfall is one of the main input into the hydrological system. It is its space-time distribution, which mainly influences the formation of runoff in the catchments outlet. Therefore enhancement of the estimation accuracy of incoming rainfall volume significantly improves simulated discharges (Ball, 1994). Comparative analysis of design rainfalls with different time distributions showed their impact on peak discharges (Ball, 1994). In order to estimate runoff sensitivity to the temporal and spatial rainfall pattern Ogden and Julien, (1993) used raster oriented rainfall-runoff model linked with the stochastic precipitation model. Rainfall event moving in the direction of flow produces higher peak than storm moving in opposite direction (Ngirane-Katashaya et al., 1985). "Sensitivity analysis of physiographic parameters is a simplified representation of a complex system. It simulates some but not all the characteristics of the system"- (Singh, 1997). Conceptual models have been evolved in surface hydrology which simulate the catchments behavior through conceptual element e.g. linear reservoirs, nonlinear reservoirs, linear channels and also through their combinations. Some of the conceptual models which are found more useful for application to small watersheds are:

---

* *Lecturer, Department of Natural Science, UIU*
** *Lecturer, EEE, Eastern University*
*** *Lecturer, Department of Arts and Science, Ahsanullah University of Science and Technology*

Clark,1945; Nash, 1960; Mathur,1972; and Pederson, 1980. Most of the conceptual models are directly or indirectly related to the theories of 'Unit Hydrograph' and 'Instantaneous Unit Hydrograph'. "Sensitivity analysis of physiographic parameters is a simplified representation of a complex system in which the behavior of the system is represented by a set of equations, perhaps together with logical statements, expressing relations between variables and parameters" (Clarke, 1973). The relationship between watershed response and its parameter can be studied suitably with the help of sensitivity analysis of physiographic parameters. It has rightly been stated that "any mathematical model formulated to represent a process or phenomenon will be based upon the extent to which it can be or has been verified" (Overton and Meadows, 1976). Kinematics waves defined as the study of motion without the influences of mass and forces; whereas dynamics is defined as the study of motions in which these influences (mass and force) are included. But certain characteristics of a watershed can make kinematics waves a dominant characteristic of that flood event (DeVriesm and Macarthur, 1979) by which a sensitivity analysis of physiographic parameters can be evaluated, the Lumped Physiographic parameter values of the study are presented in Table A1 in Appendix.

2. Model Formulation

A hydrologic model is an important tool for estimating and organizing quantitative hydrologic information. The main objectives for the development of a suitable surface hydrologic model are to study the movement of overland, (i.e. through its surface runoff) as well as stream flow components of the hydrologic cycle. The present study is aimed at developing mathematical models based on kinematics wave theory to find the sensitivity analysis of physiographic parameters by Kinematic Wave equation. The hydrodynamic theory for incompressible fluid flows gives the following set of equations (Navier-Stokes' equations):

$$\rho(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z})=X-\frac{\partial P}{\partial x}+\mu\nabla^2 u \qquad (1)$$

$$\rho(\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z})=Y-\frac{\partial P}{\partial y}+\mu\nabla^2 v \qquad (2)$$

$$\rho(\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z})=Z-\frac{\partial P}{\partial z}+\mu\nabla^2 w \qquad (3)$$

and, continuity equation : $$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \qquad (4)$$

The above four equations describe theoretically the fluid flow in any situation. Keeping in view the difficulties involved in the application of these equations for the flow of water in a channel, the following one dimensional hydrodynamic equation were suggested by (St. Venant, 1871):

$$\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = q \qquad (5)$$

$$\frac{1}{g}\frac{\partial u}{\partial x} + \frac{u}{g}\frac{\partial u}{\partial x} + \frac{\partial y_0}{\partial x} + s_f - s_0 = 0 \qquad (6)$$

Kinematic Wave equation for overland flows can be written as:

$$\frac{\partial y_0}{\partial t} + \frac{\partial q}{\partial x} = i_e = i - \Phi \text{ here } \quad q = \alpha_0 y_0^{m_0} \qquad (7)$$

where α_0 and m_0 are kinematics wave routing parameters which are directly related to conveyance of particular surface (i.e. to the slope and its roughness), q is discharge per unit width of overland flow, y_0 is the mean depth and i_e is the rainfall excess intensity [precipitant (i)–infiltration (Φ)]. For the channel flows, the kinematics wave equations can be written as follows:

$$\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = q \text{ here } Q = \alpha_k A^{m_k} \qquad (8)$$

here α_k & m_k are the kinematics wave routing parameters which are directly related to the watershed and the channel flow characteristics (a particular channel cross-sectional shape, channel slope and roughness). For the trapezoidal geometric shapes, it is not possible to derive a single simple relationship for determining α_k and m_k explicitly. An indirect approach is adopted. The Manning's equation and KW equation are employed together to compute the values of parameters α_k and m_k. The Manning's equation for discharge (Q) in a channel is given by

$$Q = \frac{1}{n} S^{\frac{1}{2}} A R^{\frac{2}{3}} \qquad (9)$$

The KW equation for channel flow is given by equation (8) and reproduced as under:

$$Q = \alpha_k A^{m_k} \qquad (10)$$

3. Analytical Solution Techniques

Analytical methods for solving partial differential equations to find the sensitivity analysis of physiographic parameters by Kinematic Wave equations are usually restricted to linear cases with simple geometric and boundary conditions. A number of researchers have developed exact as well as approximate analytical solutions for the kinematics flow approximations to compare the runoff from planes of different types and forms (Wooding, 1965; Parlangem et al., 1981). However, numerical techniques are more rational when compared to the exact and approximate analytical solutions. In principle, the method of finite differences can be applied on to nonlinear mathematical models but consistency, stability and convergence are more difficult to prove (Noye, 1982). For solving the St. Venant and Kinematics Wave equations only a few researchers have used the FEM. The finite element methodology is quite rigorous and a tedious method Stephenson et al. (1986) has used this technique for flood routing in channels and natural streams.

3.1 Approximation Kinematic Wave equations through finite differences

The discharge if any point (x) at a given instant (t) is written as $Q(x,t)$. If Q possesses a sufficient number of partial derivatives, then at the two points (x,t) and ($x+\nabla x, t+\nabla t$), the values of Q are related by the Taylor's series expansion can be written for $Q_{i-1,j}$ and $Q_{i+1,j}$ about the central value $Q_{i,j}$ respectively as:

$$Q_{i-1,j} = Q_{i,j} - \nabla x \frac{\partial Q}{\partial x} + \frac{(\nabla x)^2}{2!}\frac{\partial^2 Q}{\partial x^2} - \frac{(\nabla x)^3}{3!}\frac{\partial^3 Q}{\partial x^3} + \frac{(\nabla x)^4}{4!}\frac{\partial^4 Q}{\partial x^4} - - - -$$

$$Q_{i-1,j} = Q_{i,j} + \nabla x \frac{\partial Q}{\partial x} + \frac{(\nabla x)^2}{2!}\frac{\partial^2 Q}{\partial x^2} + \frac{(\nabla x)^3}{3!}\frac{\partial^3 Q}{\partial x^3} + \frac{(\nabla x)^4}{4!}\frac{\partial^4 Q}{\partial x^4} + - - -$$

All partial derivatives $\frac{\partial Q}{\partial x}, \frac{\partial^2 Q}{\partial x^2}, \frac{\partial^3 Q}{\partial x^3}$ - - - etc. are evaluated at the grid point (i , j).

Thus at the grid point (i, j) the partial derivatives of Q in the forward and the backward finite difference forms are written as:

$$\frac{\partial Q}{\partial x} \approx \frac{Q_{i+1,j} - Q_{i,j}}{\nabla x} \text{ and}$$
$$\frac{\partial Q}{\partial x} \approx \frac{Q_{i,j} - Q_{i-1,j}}{\nabla x} \tag{11}$$

Similarly, the first order partial derivatives $\frac{\partial Q}{\partial t}$ can be written in forward and backward finite difference forms at the grid point (i , j) as below:

$$\frac{\partial Q}{\partial t} \approx \frac{Q_{i+1,j} - Q_{i,j}}{\nabla t} \text{ and}$$

$$\frac{\partial Q}{\partial t} \approx \frac{Q_{i,j} - Q_{i,j-1}}{\nabla t} \tag{12}$$

Likewise, the first order partial derivatives of cross sectional area (A) are approximated through the forward and the backward finite differences at the grid point (i , j) as under

$$\frac{\partial A}{\partial x} \approx \frac{A_{i+1,j} - A_{i,j}}{\nabla x}, \frac{\partial A}{\partial x} \approx \frac{A_{i,j} - A_{i-1,j}}{\nabla x},$$

$$\frac{\partial A}{\partial t} \approx \frac{A_{i,j+1} - A_{i,j}}{\nabla t}, \frac{\partial A}{\partial t} \approx \frac{A_{i,j} - A_{i,j-1}}{\nabla t} \tag{13}$$

Equations (11) through (13) have been used for the finite difference approximation of the partial derivatives appearing in the final form of KW equations. Suitable computational schemes are needed for the solution of these equations.

3.2 Different Schemes For The Solution of Kinematic Wave Equations

There are three different kinds of computational schemes as: scheme I(Forward-in-time and backward–in-space), scheme II (Backward-in-time and forward-in-space) and scheme III (Backward-in-time and space) as shown in Figure 2, which have been used in this work for the solution of KW equations when applied to different watersheds under investigation.

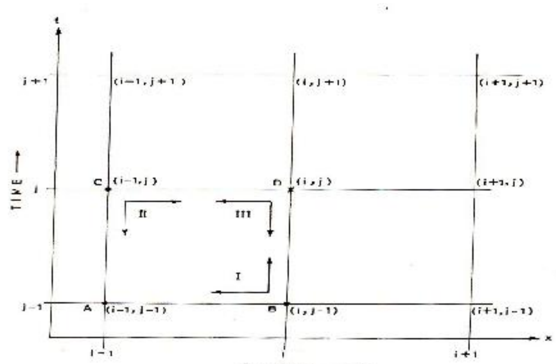

Fig 2 : Different types of schemes

It may be concluded that for ascertaining the stability of these computational schemes the Courant number has to be computed which required estimation of KW celerity values. Computation scheme I for solution of Kinematic Wave equations is given by,

$$Q_{i,j} = \alpha \left(A_{i,j}\right)^{m} \tag{14}$$

For the channel flows, q is the input due to overland flows. The parameters α and m replaced with α_k and m_k respectively. Computational scheme II for solution of Kinematic Wave equations is to be determined by,

$$A_{i,j} = \left(Q_{i,j} \Big/ \alpha \right)^{1/m} \tag{15}$$

4. Sensitivity Analysis of Physiographic Parameters by Kinematic Wave Equation

It is of common knowledge that a number of physiographic parameters interact with the input rainfall function to produce the response i.e. the runoff. This interaction makes the rainfall runoff process quite complex in nature. In different watersheds, the role of different physiographic parameters i.e. shape, soil type, land use soil cover, overland roughness, channel roughness, overland slope, channel side slope etc. may be different (Hossain M.M., 1989). Therefore, there is a need to conduct a sensitivity analysis for various physiographic parameters to ascertain the most effective. An inspection of KW equations for its overland phase (Equations 7 and 8) and for the channel phase (Equations 8 and 13) suggests that the two dependent functions i.e. discharge per unit width (q) and discharge (Q) at outlet will be functions of the following physiographic parameters

$$q = q(N, S_0, t)$$

and, $$Q = f(N, S_0, n, S, B, Z, y_c, t)$$

The method of graphical is used to asses the impact of individual parameters on the response function i.e. the discharge. In this method, one parameter is varied while others are kept constant. The various response functions values so obtained become an index for the effectiveness of the parameter.

4.1 Overland flow routing

The overland flow is in the forms of sheet flow. A unit width of the plane has been considered for the computational aspects of the runoff generation. Overland flow routing depend on different kinds of parameters such as roughness (AN), slope (So), Frude's number (FR2) etc. Overland flow for different values of overland roughness (AN) and fixed value of overland slope are shown in the figure 3:

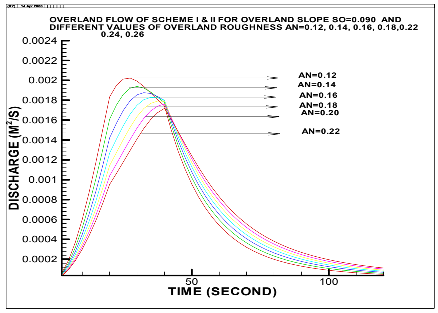

Fig 3 : Overland flow for different values of overland roughness and fixed value of overland slope.

From the above figure we observe that if the overland roughness varies then the flow routing changes quickly. So the channel slope is sensitive. Besides Overland flow for different values of overland slope (So) are shown in figure 4 and the effect of overland slope is shown in table A2 in the appendix:

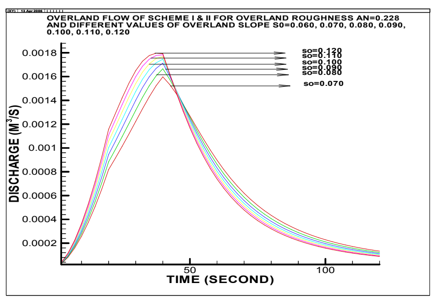

Fig 4: Overland flow for different values of overland slope and fixed value of overland roughness.

From the above figure we observe that if the overland slope varies then the flow routing changes quicker than overland roughness. So the overland slope is sensitive.

4.2 Open channel flow routing

An open channel is a conduit in which water flows with a free surface. Classified according to its origin a channel may be either natural or artificial. Natural channels include all watercourse that exist naturally on the earth, varying in size from tiny hillside rivulets through brooks, streams, small and large rivers, to tidal estuaries. Underground streams carrying water with a free surface are also considered natural open channels. The hydraulic properties of natural channels are generally very irregular. In some cases empirical assumptions reasonably consistent with actual observations and experience may be made such that the conditions of flow, in these channels become amenable to the analytical treatment of theoretical hydraulics. A comprehensive study of the behavior of flow in natural channels requires knowledge of other fields, such as hydrology geomorphology sediment transportation etc. It constitutes, in fact a subject of its own as river hydraulics. Open channel flow routing for different value of overland roughness (AN) shown in the figure 5:

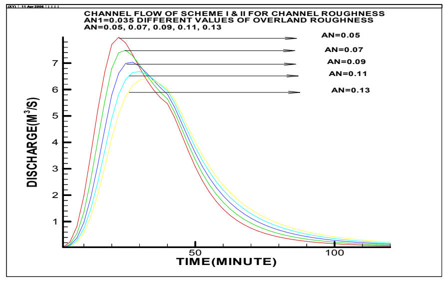

Fig 5 : Open channel flow routing for different value of overland roughness (AN).

The above figure shows that the effect of overland roughness on channel flow is sensitive, because if the overland roughness changes the shape of channel flow is changed. Open channel flow routing for different value of channel roughness (AN1) is shown in the figure 6 and the effect of overland roughness is shown in table A3 in appendix:

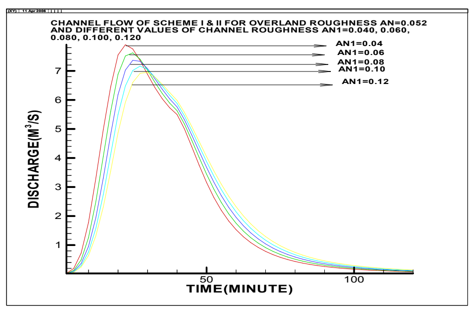

Fig 6 : Open channel flow routing for different value of channel roughness (AN1).

The above figure shows that the effect of channel roughness on channel flow is sensitive but is not like overland roughness. So overland roughness is very sensitive on channel flow. Open channel flow routing for different value of side slope (Bz) is shown in the figure 7:

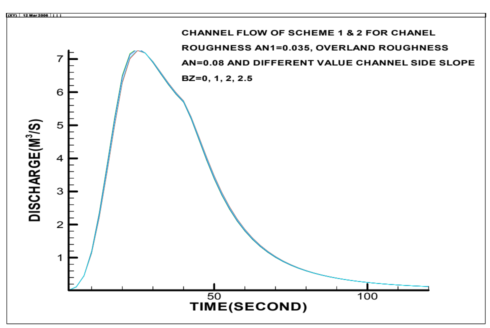

Fig 7 : Open channel flow routing for different values of side slope (Bz).

From the figure we can observe that the effect of side slope on the flow routing of channel flow is not an important factor. So side slope is not so sensitive. Open channel flow routing for different value of overland slope (So) is shown in the figure 8:

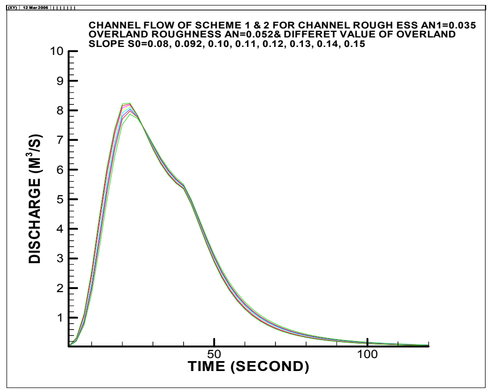

Fig 8 : Open channel flow routing for different value of overland slope.

The above figure express that the effect of overland slope on the flow routing of channel flow is not an important factor. So overland slope is not so sensitive.

5. Discussion of sensitivity analysis

Overland flow for different values of overland slope was discussed in fig: 4. From those results we observe that if the overland slope (So) varies then the flow routing is changed quicker than overland roughness (AN). So the overland slope is sensitive. In section 4.2 channel flow routing is described and we observe that channel flow routing depend on overland roughness, overland slope, channel roughness and channel side slope. Open channel flow routing for different values of overland roughness (AN) was discussed in fig: 5. From those discussions we observe that the effect of overland roughness on channel flow is sensitive, because if the overland roughness changes the shape of channel flow is changed. In fig: 6 the open channel flow routing for different values of channel roughness (AN1) is discussed. We observed that the effect of channel roughness is not as sensitive as overland roughness (AN). Channel flow routing for different values of side slope is discussed in fig: 7. From those sections we observed that the effect of side slope on the flow routing of channel flow is not an important factor. So side slope is not so sensitive. In fig: 8 channel flow routing for different values of overland slope is described and observed that the effect of overland slope on the flow routing of channel flow is not an important factor. So overland slope is not so sensitive. From the above discussions we come to the conclusion that the channel roughness, overland roughness and overland slope are sensitive.

6. Conclusion

The results of the KW theory applications to three natural hilly watersheds and one agricultural watershed are discussed and sensitivity analysis of different kinds of channel and overland parameter. Suitable conclusions have been drawn from these discussions. Finally we tried to discuss the channel flow routing and overland flow routing besides the effect of overland slope and overland roughness as shown in table A2 and table A3 in the following appendix. The computer program for flow routing is developed in Lahey ED Developer. From that section we observed that the flow routing depend on different kind of channel and overland flow parameters. Besides overland flow for different values of overland roughness was discussed and we observe that if the overland roughness varies then the flow routing is changed quickly. From those results we come to the conclusion that the channel slope is sensitive.

References:

Ball J.E., (1994): The influence of storm temporal patterns on catchment response. J. Hydrol., *158*, pp. 285−303.

Ogden F.L., Julien P.Y., (1993): "Runoff sensitivity to temporal and spatial rainfall variability at runoff plane and small basin scales". *Water Resource Research*, *29*, pp 2589−2597.

Ngirane-Katashaya G.G., Wheater H. S., (1985): "Hydrograph sensitivity to storm kinematics". *Water Resource Research*., *21*, 3, 337−345.

Singh V.P., (1997): "Effect of spatial and temporal variability in rainfall and watershed characteristics on stream flow hydrograph". *Hydrological processes*, 11, pp 1649−1669.

Clarke, R.T (1973). "Mathematical Models in Hydrology. Food and Agri. Organ. of the United Nations", *Irrigation and Drainage paper* No. 19, Rome.

Clark, C.O.(1945). "Storage and the Unit Hydrograph". Proceedings. ASCE, Vol. 69:1333-1360.

Dooge, J.C.I. (1969). "A General Theory of the Unit Hydrograph". *Journal of Geophysical Research*, AUG, 64(2): 241-256.

Mathur, B. S. (1972). "Runoff Hydrographs for Uneven Spatial Distribution of Rainfall" Ph.D. thesis, The Indian Institute of Tech., New Delhi, India.

Pederson, J. T., Peters, J.C. and Helweg, O.J. (1980). "Hydrographs by Single Linear Reservoir Model". Journal of Hydro Division. ASEC, Vol.106 (HY5): pp 837-652.

Overton, D.E. and Meadows, M.E. (1976). *Storm water Modeling*. Academic Press, New York.

DeVriesm J.J., and Macarthur, R. C. (1979). "Introduction and Application of Kinematics Wave Routing Techniques Using HEC-1". Training Doc. No. 10, HEC, DAVIS, USA.

Wooding, R.A., (1965). "A hydraulic Model for the Catchments –Stream Problem". (a) I. KWT, (b) II. *Numerical Solutions. Journal of Hydrology*, Vol. 3(3): pp 254-282.

Parlangem J.Y., Rose, C.W. and Sander, G.C. (1981). "Kinematics Flow Approximation of runoff on a plane: An exact analytical solution". *Journal of Hydrology*, Vol. 52: pp 171-178.

Moore. I.D. et al. (1985). "Kinematics Overland Flow: Generalization of Rose's approximate Solution". *Journal of Hydrology*, Vol. 82: pp 233-245

Noye, J. (1982). *Numerical Solutions of Partial Differential Equations. North*- Holland Publishing Company, Amsterdam.

Hossain, M. M. (1989). "Application of Kinematic Wave Theory to Small Watersheds" Ph.D. Thesis. Submitted in the Roorkee University, UP, India

Stephenson, D., and Meadows, M.E (1986). *Kinematics Hydrology and Modeling. Elsevier, Amsterdam.*

Appendix

Table A1 : The Lumped Physiographic Parameter values:

Sl. No.	Particulars	Unit
(1)	(2)	(3)
1.	Area	82.0 hectares
2.	**Overland(Plane):**	
	(a) Average length (each side)	248.0 meters
	(b) Average Slope (each side)	0.092
3.	**Channel :**	
	(a) Average length	1650.0 meters
	(b) Average Slope	0.072
	(c) Average roughness	0.035
	(d) Average bed width	3.0 meters
	(e) Average side slope	2.5 H:1 V

Sensitivity of (i) Overland Slope and (ii) Overland Roughness:

Table A2 : Effect of Overland Slope:

Sl. No.	Overland Slope	Percentage (%)	Peak (m^3/s)	Time to Peak (min)	Volume (m^3)*100
(1)	(2)	(3)	(4)	(5)	(6)
1.	0.046	50.0	3.329	50.0	111.16
2.	0.069	75.0	3.500	45.0	112.79
3.	0.092	100.0	3.642	42.5	113.662
4.	0.115	125.0	3.756	40.5	114.22
5.	0.138	150.0	3.849	37.5	114.62

Table A3: Effect of Overland Roughness :

Sl. No.	Overland Roughness	Percentage (%)	Peak (m^3/s)	Time to Peak (min)	Volume (m^3)*100
(1)	(2)	(3)	(4)	(5)	(6)
1.	0.070	50.0	4.315	32.5	116.08
2.	0.105	75.0	3.938	37.5	114.95
3.	0.140	100.0	3.642	42.5	113.662
4.	0.175	125.0	3.434	47.5	112.18
5.	0.210	150.0	3.260	52.5	110.61